

Effects of Defect Size and Number Density on the Transmission and Reflection of Guided Elastic Waves

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Abstract

We explore the scattering of guided elastic waves in plastic cylinders by wavelength-sized defects. A random field of defects is introduced into the cylinder in a localized region, a photoacoustic source generates elastic waves on one side of the damaged region, and then two ultrasound transducers measure the amplitude of the waves transmitted and reflected by the damaged region. We found that for defects approximately 3λ in diameter that the transmission amplitude decays exponentially in the volumetric number density of defects. Furthermore, we discovered that for a constant defect density, in the Mie regime, there is not statistically significant transmission amplitude variation between different defect sizes. Additionally, in the Mie regime, the reflected wave due to backscatter is too small to be detected with our equipment.

Introduction

In the early 2000's, a number of groups conducted experiments using acoustic non-destructive evaluation (NDE) techniques to investigate musculoskeletal damage. Among the damage modes investigated were osteoporosis and stress fracture. The outlook for the field following these experiments was largely positive. However, no major efforts to use NDE for musculoskeletal damage detection have been documented since (c. 2006).

In this paper, we address stress fractures specifically as we investigate the feasibility of using this sort of measurement for stress fracture detection. Part of the reason that characterizing stress fractures is challenging is that the literature has a confused, often contradictory medley of biomechanical definitions of stress fracture. Some propose that a stress fracture is a field of random microcracks in the bone. Others posit that stress fractures are a collection of cell-sized cavities. Still others characterize it by delamination of planes of bone material.

The goal of this experiment was to investigate whether the microcrack model of a stress fracture could yield detectable information using ultrasonic NDE and photoacoustic excitation.

Methods

For this experiment, which was designed for the microcrack model of a stress fracture, we simply modeled our microcracks as macroscopic, drilled holes in our bone phantom, 12" PVC tubing. We varied both the size of the holes and also the density of the holes within the damaged region. The goal of these variations was to establish a trend in the reflection and transmission coefficients as a function of these two parameters which we could eventually use to extrapolate to the case of microcracks.

The bone phantom was 12" long segments of PVC tubing. The tubing was 3/4" in diameter, and had 1/8" thickness (similar in dimensions to the cortical bone of the tibia). We then defined a 1 1/2 sq. in. region on each of the pipes for damage. In this region we drilled holes of diameters 1/8" through 1/32" in the different samples. The effective density of damage, D_{eff} , was defined in terms of our largest hole size, 1/8", and the number of holes drilled:

$$D_{eff} = N \left(\frac{Diameter}{1/8"} \right)^2$$

That is, the amount of material removed by each hole drilled goes like the square of the diameter. Thus, in order to study the effect of hole size on transmission and reflection at constant density, we need to drill more holes when we are using the smaller drill bits.

Our experimental apparatus consisted of two 1MHz Olympus transducers situated on either side of the damaged region. We chose to measure both shear waves and compression waves for our experiment. The shear transducer used was the V153, and the compression transducer was the V183.

While the transducers act as receivers, we used a photoacoustic source: a Q-switched laser (Panther OPO) operating at 1.55um and with a pulse width of 7ns, a repetition rate of 30Hz and an average power of 65mW. This configuration seems quite successful at exciting elastic waves in the tube at 1MHz. A complete diagram of the apparatus is given in Figure 1.

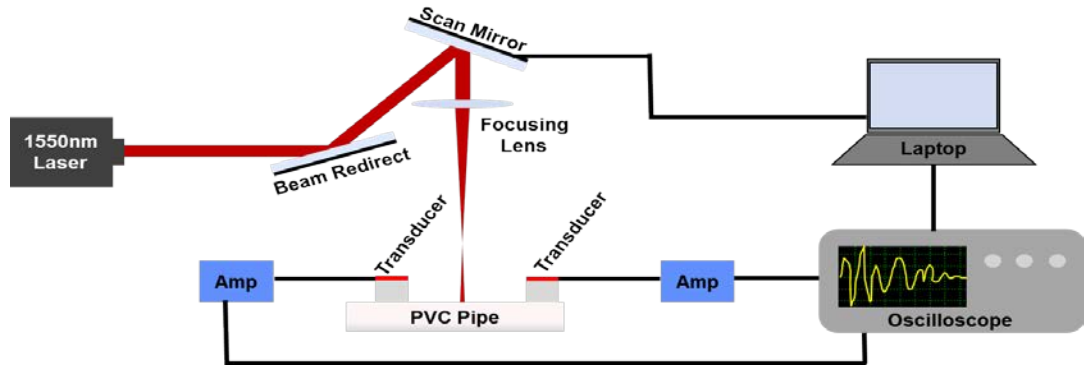


FIG. 1: The pulsed laser is directed to and focused near the surface of the PVC pipe, where elastic waves are generated and then picked up by the two Olympus V153 transducers. The signal is amplified by an Olympus 5072PR before being readout at a Tektronix TDS 2024B oscilloscope which is then recorded into a laptop using a Labview program. The scan mirror is for future experiments where we may vary the laser spot position.

Data & Analysis

The data acquired from the oscilloscope trace gives the amplitude of the shear (compression) at the location of the transducer as a function of time after the laser fires. In order to determine the effect of reflection and transmission, we take our data from the control run, and compare the data collected on subsequent runs to it by subtracting it off and then normalizing by it to give a relative change.

What becomes immediately obvious is that, despite the best efforts to maintain an equal distance between the transducers and laser spot between runs, there will be small variations. This variation amounts largely to a shift in the zero of time. Therefore, we do a cross correlation between the control signal and the signal obtained in each trial, and then apply a shift to the trial signal which maximizes the correlation. It is with this shifted trial signal from which we compute the reflection (transmission data). This process is outlined in Figure 2.

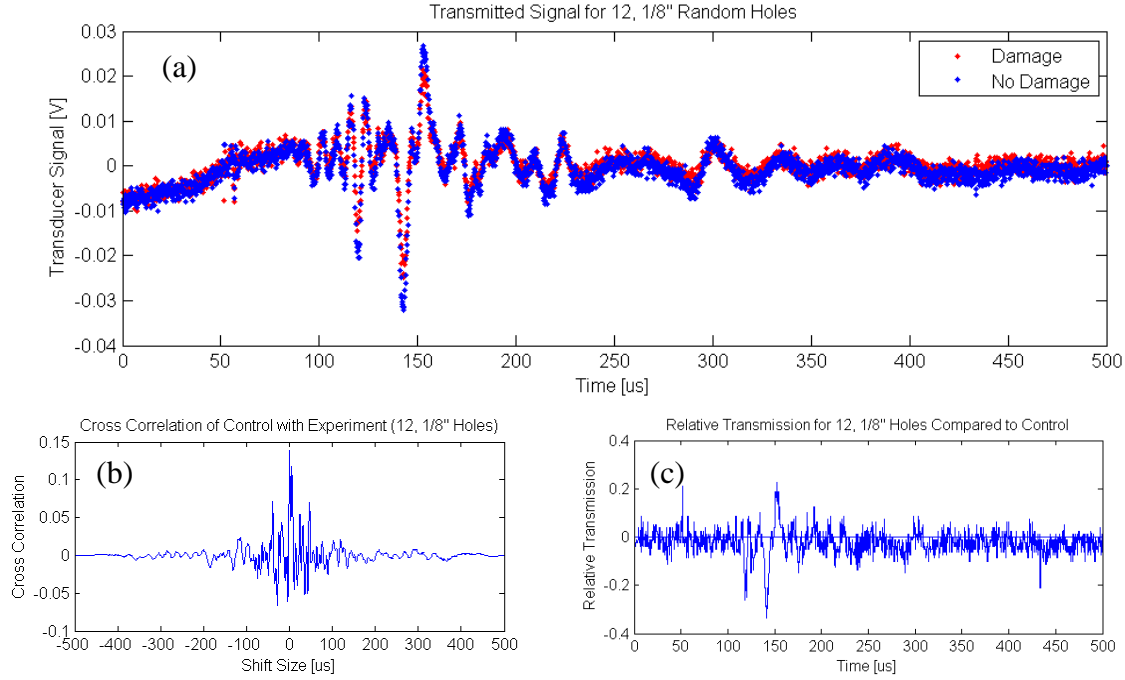


FIG. 2: Here we outline the process for analyzing transmission data. (a) We have a control signal (blue) which indicates the amplitude of shear at the transmission transducer as a function of time (the laser fires at 50us). Note that for transmission, the signals are nearly identical, except that the damaged sample exhibits a decrease in amplitude. (b) We do cross correlation between the two signals to account for any unintended offsets between the transducer and the laser spot. Here the correlation is maximized for a shift of 0.2us. (c) We subtract off the control signal from the damage signal, then normalize by the control. In for the most positive peak, we see that the transmission is 22.5% (positive signal) different from the control at 152us, and 33.8% (negative signal) different at 141us. This corresponds to a transmission coefficient of between 0.662 and 0.775, a significant result. N.B. Ultimately, this dataset was rejected after we found the spacing between the laser spot and the transducers to be too short, causing interferences between waves which disperse after longer distances.

There was one particular case where the cross correlation method was insufficient for transmission. For the case of $D_{\text{eff}}=16$ (the highest effective density) with the largest holes, there was an appreciable phase shift in the data of 4us. This size phase shift was too large for the cross correlation to correct for, so we manually aligned the oscillations in the vicinity of 200us, as can be seen in Figure 3.

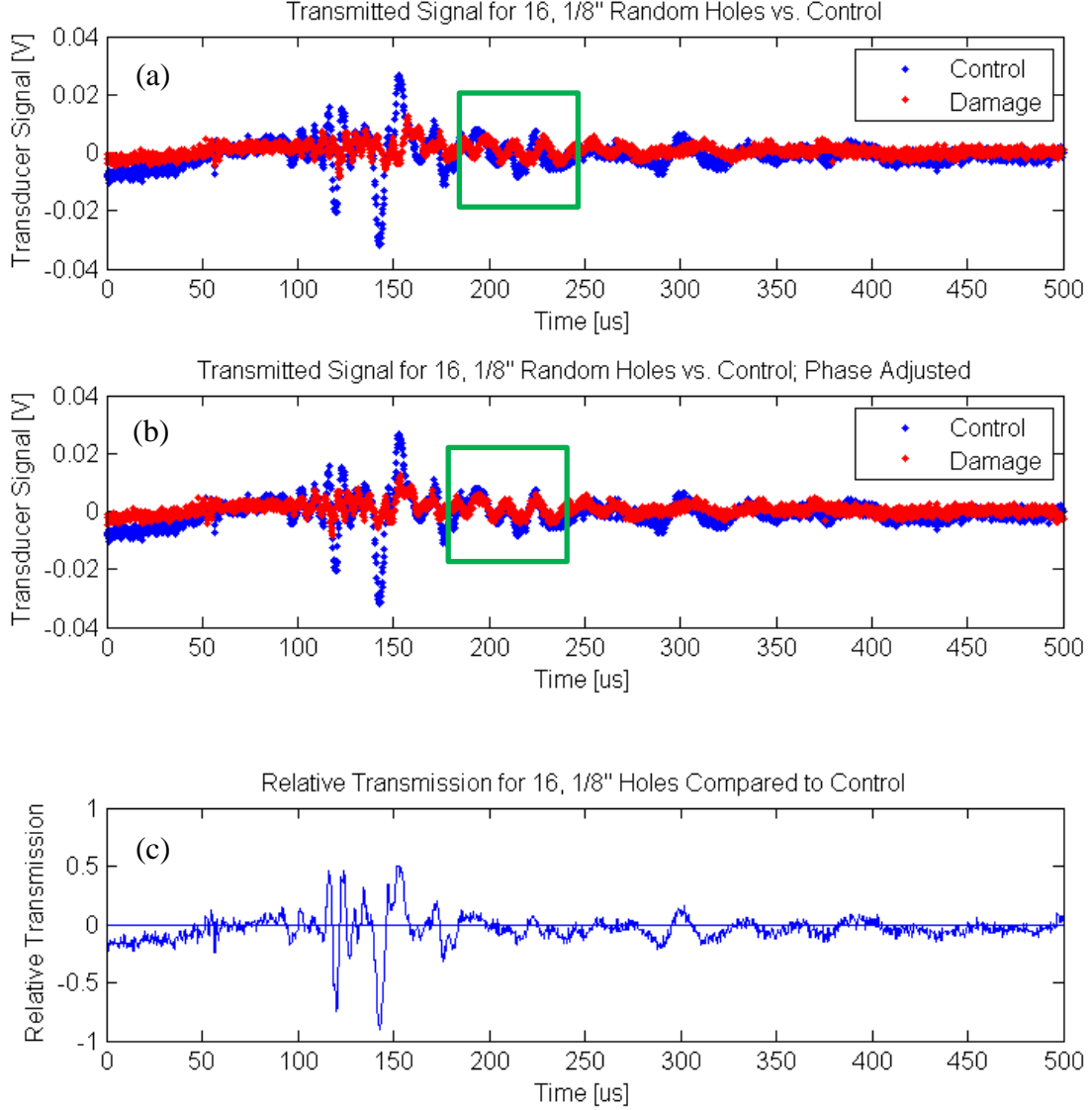


FIG. 3: (a) Here the trial data is overlaid on the control data. The boxed region demonstrates a large phase shift between a common feature of the two datasets in the region between 200-250us. (b) After adjusting for this 4us phase manually we find that the peaks and troughs of the data generally line up, which allows us to find the transmission amplitude. (c) The relative change in transmission for this signal was 90.0% (negative peak) at 142us (which is within 1us of the negative peak from the dataset from Figure 3), while it is 50.6% for the positive signal at 152us. This means that the transmission amplitude for the wave arriving at 142us is 0.1. This feature has a transmission coefficient most strongly affected by the density of holes.

We repeat this process for three more D_{eff} values, so that our total experimental range covers $D_{\text{eff}} = \{4, 6, 8, 12, 16\}$. We fit the data with an exponentially decaying relationship, as might be expected for turbid media. The least squares fit gives an amplitude of very nearly 1, in accordance with the requirement that in the absence of any scattering bodies, the transmission

should be perfect. The point values were taken from the extrema of the 141us peak, and the error bars are equal to half the typical difference between the extreme point and its neighbors, which comes out to be ± 0.075 . This measure of the error accounts for the fact that the extreme point could have been even larger than normal after normalization if the control data point that contributed to the extreme point was particularly small. Our assumption is that the extreme point still probably lies above its nearest neighbors, but not necessarily at its exact value. With this choice for measuring our error we obtain a $\chi^2=1.72$. Since we utilized three degrees of freedom, our χ^2 ought to lie between 0.55 and 5.45.

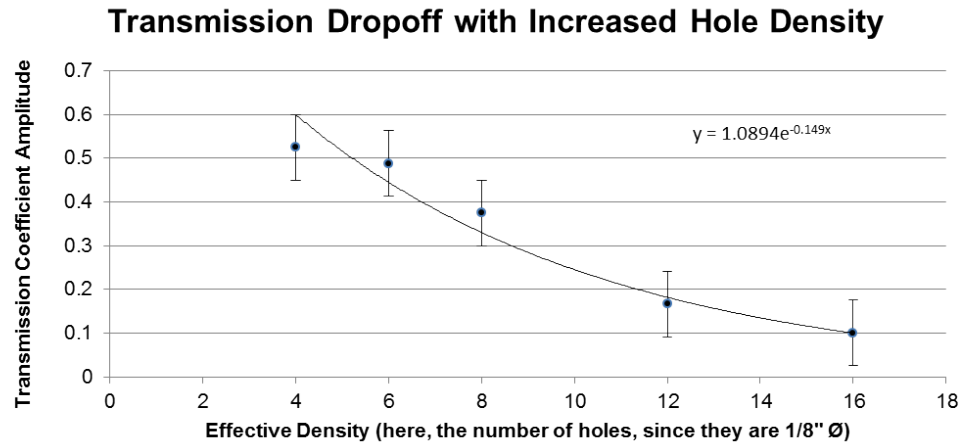


FIG. 4: Within statistical uncertainty, the effect of hole density on the transmission amplitude seems to obey an exponential law, in accordance with our intuition about turbid media.

Discussion & Conclusions

It is worth noting that the cross correlation correction just described works quite well for the transmission signal. However, this process needs a slight modification for the reflected signal. The reflected signal is really the sum of two signals: the direct waves from the laser spot which reach the transducer before the round trip of the reflected waves, and also the reflected waves. If we had a monochromatic source then these two signals would be quite far separated in time, and analysis would be simple. However, the thermal, photoacoustic conversion process is inherently broadband. Therefore, since we are dealing with wavelengths on the same length scale as the pipe thickness, we see a family of highly dispersive guided modes. This dispersion leads to spreading of the signals in the tube. This spreading causes the slowest components of the

direct waves to interfere with the fastest components of the reflected waves. As a result of this overlapping, it becomes very challenging to separate out the reflected wave from the much higher-amplitude direct wave. Separating the transducer from the laser spot to allow the slower waves to propagate past the transducer before the fastest reflected waves arrive creates an attenuation distance that makes detecting the reflected waves technically impossible to detect.

Furthermore, in the regime we have where the defects are of the same order as the wavelength of the ultrasound, we find ourselves confronted with Mie scattering, which has weaker backscatter compared to forward scatter, making our reflected signal even lower compared to that of the transmission signal.

We also do not have the luxury of relating the reflection and transmission amplitudes via

$$|R|^2 + |T|^2 = 1$$

because we lose a good deal of our signal as it scatters sideways out of the damaged region.

One issue worth addressing from the analysis is the necessity of the cross correlation processing. While shifting after cross correlation allows us to derive amplitudes from the collected data, it does so by eliminating information about the phase of the wave. For instance, in the case of the 16, 1/8" holes with the 4us phase shift, that phase information could have been a valuable asset to someone working on the inverse scattering problem. However, with our current apparatus we do not know whether the phase shift was due to scattering or a longitudinal offset of the transducer. Therefore, we cannot include phase in our discussion of scattering at all. A future experiment might address this by somehow fixing the transducers to the optics for precise, consistent alignment between samples.

In the future, we need a method for addressing all of these shortcomings. The most essential adjustment is to leave the regime of Mie scattering in favor of the Rayleigh scattering regime, which is physically more similar to the microcrack model. One possible way of accomplishing microcracks in a ceramic material would be via exposure to hydrofluoric acid vapor. Secondly, we need to find more sensitive methods for measuring the reflected wave. Ideally, this could be accomplished by using a monochromatic acoustic source so that we do not suffer from the dispersion from the waveguide nature of our cylinder. Using a monochromatic source would allow us to more effectively time gate to neglect the direct wave.

In conclusion, we observed that in the Mie regime, the volumetric volume density of defects causes an exponential dropoff in transmission signal, while the defect size is a much less

sensitive parameter. Additionally, we found that it is much simpler to make measurements on transmission than reflection in our current configuration.